

Lecture 10

Penetration Mechanics I

(references are in lecture 11 notes)

1. Rigid Penetration of Targets

Projectiles may be rigid or deforming. Rigid projectiles include nails, hard bullets, and low velocity penetration of fluids and soils. Deforming projectiles include bullets hitting hard armor, warhead-launched fragments and the fragment simulating projectiles (FSP), usually used as surrogate long rods designed to penetrate heavy armor, and shaped charged jets (which are produced by explosive collapse of ductile metal cones).

The physical processes happening in the projectiles are separate from those occurring in the target, but are constrained by the requirement that stress be continuous across the projectile-target boundary and that displacements be compatible at the boundary.

For example, figure 1 illustrates a Russian anti-tank projectile designed to take advantage of rigid body penetration. It contains a hard brittle WC core, a WHA cap designed to provide confining stress at impact, and a steel shank to push the core. U.S. anti-tank projectiles are long rods of $L/D=32$ which are made from depleted uranium.

Armor piercing bullets (AP) are also designed to achieve rigid body penetration. Figure 2 illustrates penetration data for an AP (armor-piercing) bullet penetrating on armor aluminum alloy. Bullets can be AP, ball (soft steel) or SLAP (saboted light armor piercing). Other examples of rigid body penetration include punches, nails, and hardness indenter.

Rigid body penetration is well described by cavity expansion models. Let σ_c be the stress required to open a cavity from zero

radius. It can be easily shown that the force required to drive an indenter into a target is $F = \rho r^2 \sigma_c$ regardless of the nose shape.

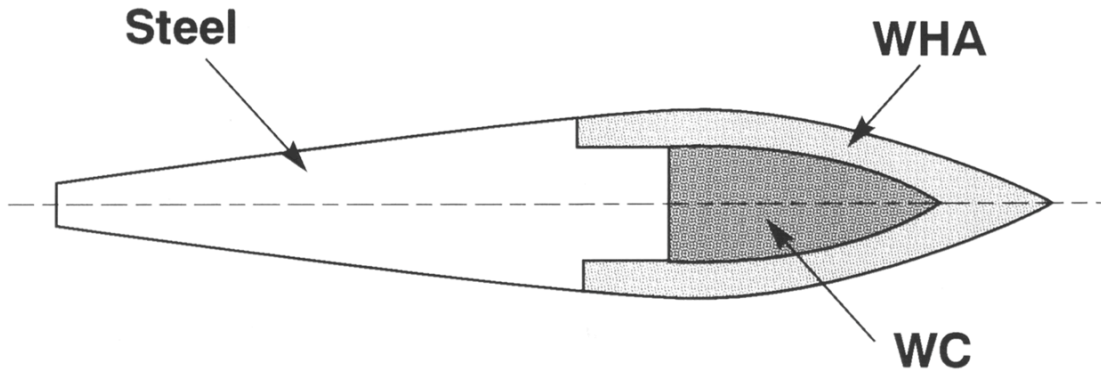


Figure 1 Soviet Penetrator

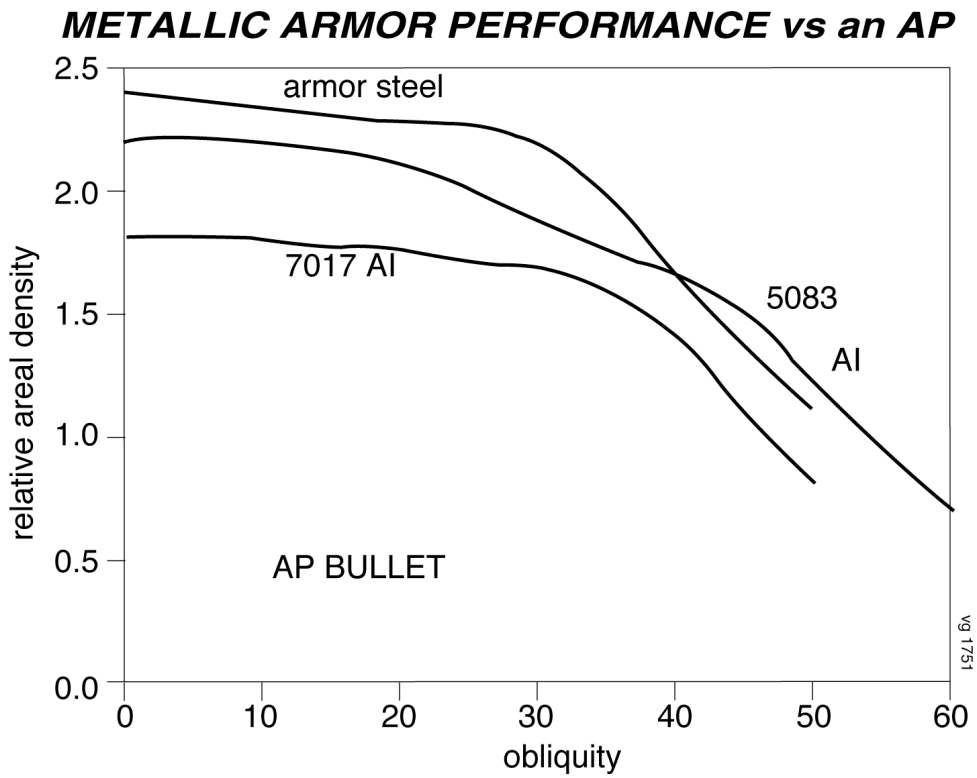


Figure 2

2 Steady Penetration (Thick targets)

Target elements may be considered relatively thin or thick. A relatively thick target is one in which the transient effects, due to initial impact and either aerostation of the projectile or breakout, are relatively small compared to the steady state. In practice, thickness is often measured in terms of projectile diameters, and, in targets that are more than 5 diameters thick, steady state effects usually dominate. Viewed from this perspective, thick target response also implies a projectile with a high L/D ratio; otherwise, the target will not be perforated. Consequently, thick target theory primarily applies to long rod projectiles.

The simplest steady penetration equation is the Poncelet equation

$$m \frac{dV}{dt} = R + \frac{1}{2} C_{D_t} A V^2 \quad (1)$$

which adds strength and drag terms and has the solution

$$P = \frac{m}{C_{D_t} A} \ln \frac{V}{V_0} + \frac{C_{D_t} A V_0^2}{2R} \left(\frac{V}{V_0} - 1 \right) \quad (2)$$

Note that for a rod, this implies that penetration mainly depends on $\rho_p L / \rho_t$.

Steady state penetration of long rods and shaped charge jets is usually described by what has become known in the West as "Tate theory" (Tate, [1]). Consider a steady state penetration as depicted in Figure 3. Here, erosion takes place in the projectile and in the target.

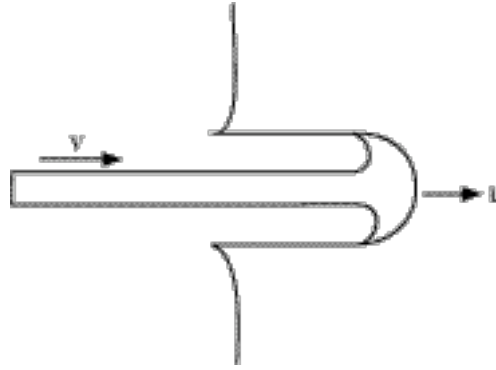


Figure 3 Steady state penetration.

We appeal to the argument that stresses on either side of the penetrators/target boundary must be the same, otherwise the boundary would accelerate. The particle velocities must be the same, too. The result is the so-called "modified Bernoulli" equation, which was assumed by Tate and has recently been derived by Satapathy [2].

$$\frac{1}{2} \rho_p (v - u)^2 + Y = \frac{1}{2} \rho_t u^2 + R \quad (3)$$

In this equation, R represents the effective strength of the target, and Y is the strength of the projectile. An important component of this theory is the notion that the projectile acts or is a Taylor impact. It slows down, but the maximum stress that it can support outside of the impact region is Y . So it decelerates according to reverberating elastic waves, each of which carries stress Y . The general solution to this equation when projectile slow-down is ignored is:

$$\frac{P}{L} = s \equiv \frac{\rho_p + \sqrt{\rho_p + \frac{1}{s_h^2} (1 - \rho_p)}}{\rho_p + \frac{1}{s_h^2}} \quad (4)$$

where $\rho = 2(R - Y) / \rho_p V^2$

At large values of T_t , this is usually an adequate approximation.

A very attractive feature of this solution is that the target is described by only two parameters: density and effective strength, R .

The target resistance, R , is equivalent to the stress required to open a cavity in a target from zero radius. The solution for cylindrical cavities was first published in a famous article by Nobel Laureates Bishop, Hill, and Mott [3]. In an elastic plastic material, the solution is:

$$R = \frac{2\sigma}{3} + \ln \left(\frac{\sigma}{3\sigma(1-\nu)} \right) \quad (5)$$

Cavity expansion solutions have been worked out for many material behavior modes and for ductile and brittle materials. See Satapathy and Bless, [4], for a general treatment. Cavity expansion can also be used to compute the rate of crater growth. [See, for example, Bless et al. [5].

Cavity expansion can also be used to compute friction effects. For the elastic plastic solution the force required to drive a conical indenter becomes

$$= \sigma r^2 R \left(1 + \frac{\mu}{\tan \alpha} \right) \quad (6)$$

Where μ is the friction coefficient and α is the nose half angle. Hence, sharp projectiles can be less effective penetrators.

The Tate equation is often used with values of R that are found empirically to give good matches to experiments. The technique is often expanded to model complex targets, even though the target response may not involve simple cavity expansion. These procedures must be used with caution.

There are many special cases to the Tate solution, for different quadrants in the Taylor diagram, as discussed, for example, in Tate [1]. The general form of the solution to the Tate equation, which applies to intermediate to high Taylor numbers, is shown in Figure 4.

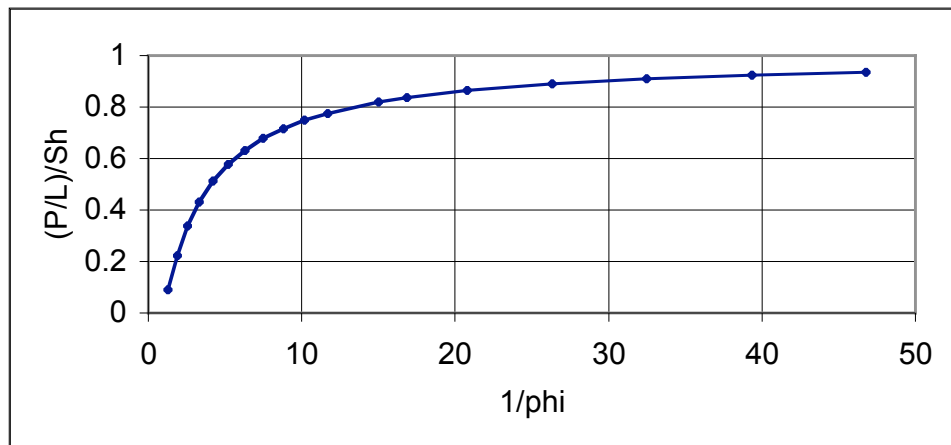


Figure 4 Tate equation solution-S curve.

This S-shaped curve shape occurs for most combinations of projectile and target materials of interest to the warhead designer. There is a threshold velocity where penetration begins. It is given approximately by:

$$V_o = \sqrt{\frac{2(R \square Y)}{L}} \quad (7)$$

Threshold velocities are relatively low. For most standard penetrators striking structural metals, they are below 1 km/s. Figure 4 gives the curve for long tungsten rods penetration armor steel.

Above the threshold velocity, penetration increases rapidly. However, eventually the projectile begins to deform, and then penetration begins to level off. In the limit of very high velocity (the upper right corner of the Taylor diagram), penetration

approaches the so-called "hydrodynamic limit." The hydrodynamic limit is given by

$$P = L S_h = L \sqrt{\rho_p / \rho_t} \quad (8)$$

The penetration at the hydrodynamic limit depends only on the density ratio and length of the projectile. It is important, however, not to generalize from this formula, because hydrodynamic penetration is seldom achieved in practice. In the region where strength is important, penetration does not usually scale as the square root of penetrator density. Also, penetration always increases as $V \propto$, contrary to the notion of an asymptote.

At high velocities there is an overshoot of the hydrodynamic limit due to "secondary penetration. There are two causes of secondary penetration. One is due to a second impact of debris in the crater. According to Tate theory penetrator material that interacts with the penetration front has velocity, $2u-v$. This is >0 (into the target) if $\rho_p > \rho_t$, and it can reset in a second impact with augmented penetration [2]. The other cause of secondary penetration is inertia in the target at the point where the penetrator is "used up".

For very strong projectiles and relatively weak targets, it is possible to substantially overshoot the hydrodynamic limit [3]. Unfortunately, there is no general formula for when projectile yielding occurs. In practice, bending of projectiles in targets often occurs before gross plastic flow. However, the rule implied by the Tate solution is approximately valid: projectiles remain rigid when

$$V < \sqrt{\frac{2(Y \propto R)}{\rho}} \quad (9)$$

In more recent work, penetration by long rods is often represented by the so-called "Odermat" equation.

$$P/L = a \exp\left[-\left(V_0/V\right)^2\right] \quad (10)$$

This is a very useful form for empirically fitting data. Figure 6 shows long rod data for steel and titanium penetration along with the Odermat parameters. Several authors have attempted to reify the Odermat parameters (for example, Gooch et al., [5]), but it is probably best to regard them as essentially fitting parameters for an empirical equation.

An improved version that takes scaling into account was purposed in [4]:

$$\frac{P}{L} = g = A \exp\left[-\left(b/v\right)^c (1 + K/D)^{c/2}\right] \quad (11)$$

where the values of the empirical parameters are given in Table 1.

Table 1. Fitting parameters for generalized Odermat equation

| Parameter | $L/D \geq 30$ WHA rods |
|------------|------------------------|
| A | 1.7 |
| b (km/s) | 1.2 |
| c | 2.9 |
| K (mm) | 3.5 |

There are several compendia of target penetration data by long rods. The most widely used has been compiled by Southwest Research Institute (Anderson et al., [6]); it is available on diskette. Figure 5 is drawn from the compendium.

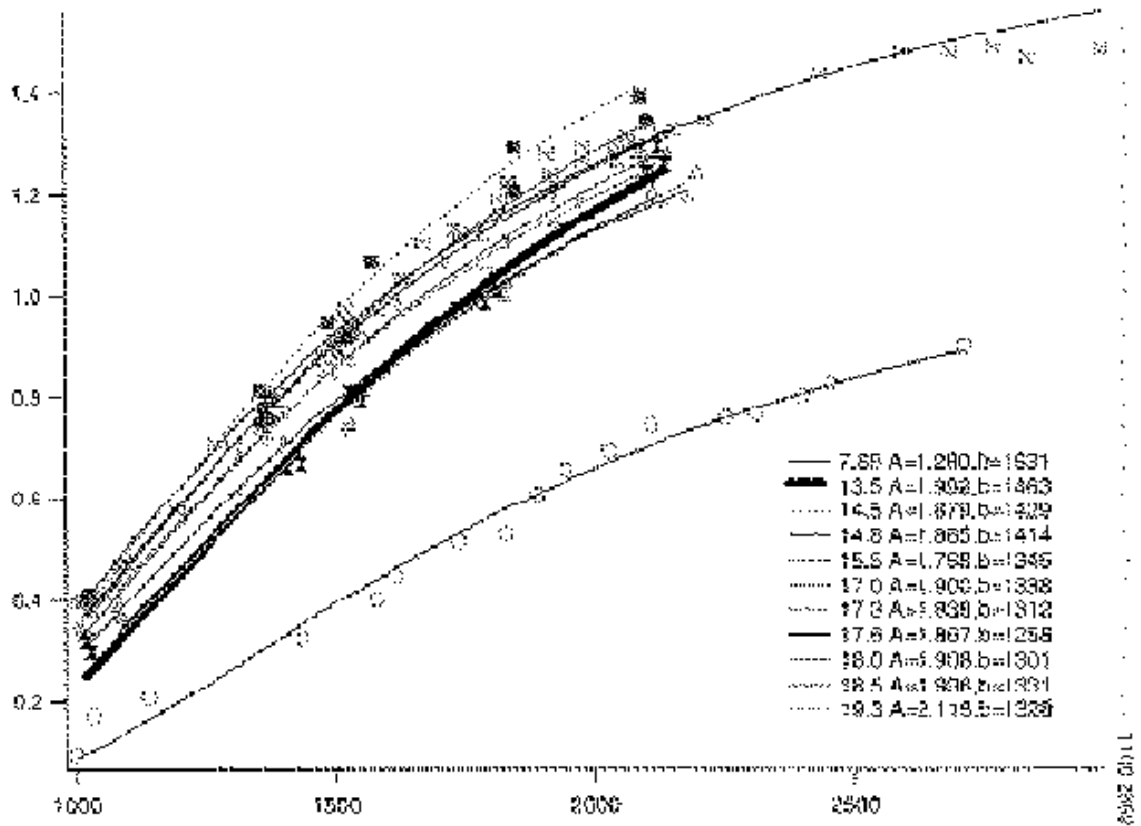


Figure 5. Penetration (P/L) of $L/D=10$ tungsten alloy rods into materials of two densities, fitted with Odermat equation ($b \equiv V_0$). Velocity in m/s.[6]